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# Delay of Vehicle Motion in Traffic Dynamics

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## Abstract

We demonstrate that in Optimal Velocity Model (OVM) delay times of vehicles coming from the dynamical equation of motion of OVM almost explain the order of delay times observed in actual traffic flows without introducing explicit delay times. Delay times in various cases are estimated: the case of a leader vehicle and its follower, a queue of vehicles controlled by traffic lights and many-vehicle case of highway traffic flow. The remarkable result is that in most of the situation for which we can make a reasonable definition of a delay time, the obtained delay time is of order 1 second.

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# 1 Introduction

The history of traffic dynamics began early in 1950s. Two different models were proposed: the one is car-following model[1, 2, 3], which describe the motion of vehicles by many-variable differential equations, and the other is the fluid dynamical model[4], which regard traffic flow something like fluid. These two models have been further investigated by many authors. However either model did not succeed in explaining the behavior of real traffic flow in some points. Especially those traditional models can not attain an unified understanding the most remarkable fact that traffic flow has two phases; one is free flow with low car-density and high average speed and the other is congested flow with high car-density and low average speed.

Recently, in these two models, there are found new mechanisms which explain the existence of two phases. In car-following models, a modified model was proposed by introducing optimal velocity[5]. (We call this ‘Optimal Velocity Model’ (OVM) hereafter.) The equation of motion of OVM is given by

$$\ddot{x}_n(t) = a\{V(\Delta x_n(t)) - \dot{x}_n(t)\} \quad n = 1, 2 \cdots N, \quad (1)$$

where the notations are; car number  $n$ , time  $t$ , position of  $n$ -th car  $x_n$  and its headway  $\Delta x_n$ . A dot on the variable denotes differentiation with regard to time  $t$ , sensitivity  $a$  is a constant parameter. The essential difference from traditional car-following models is the introduction of an optimal velocity  $V(\Delta x)$  of a vehicle, which value is a function of headway distance. A driver reacts according to the difference between the vehicle’s velocity and the optimal velocity  $V(\Delta x)$  and controls its velocity by accelerating (or decelerating) his vehicle proportional to this velocity difference. The dynamical equation of OVM has two different kind of solutions. One is a homogeneous flow solution and the other is a congested flow solution which consists of two distinct regions; congested regions (high density) and free regions (low density). In OVM, if the density of vehicles is above some critical value the traffic congestion occurs spontaneously, which can be understood as a sort of phase transition from homogeneous flow state to congested flow state[5, 6].

Also in fluid dynamical models, the following equation has been proposed[7].

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = \frac{V(\rho) - v}{\tau} - \frac{c_0^2}{\rho} \frac{\partial \rho}{\partial x} + \frac{l^2}{\rho} \frac{\partial^2 v}{\partial x^2}, \quad (2)$$

where  $c_0$ ,  $l$  and  $\tau$  are constant parameters of the system and density  $\rho$  and velocity  $v$  of vehicles are functions of location  $x$  and time  $t$ . They call the function  $V(\rho)$  ‘safe velocity’. To see the similarity of these two models, let us ignore second and third term of rhs of Eq.(2) and use  $D/Dt = \partial/\partial t + v\partial/\partial x$ ; the differentiation acting on the variables of individual vehicles. The fluid dynamical equation (2) becomes

$$\frac{Dv}{Dt} = \frac{V(\rho) - v}{\tau}. \quad (3)$$

This is quite similar to Eq.(1) with optimal velocity  $V(\Delta x)$  and sensitivity  $a$  replaced by ‘safe velocity’  $V(\rho)$  and parameter  $1/\tau$ , respectively. The reason why these two models can explain the formation of traffic congestion comes from the form of dynamical equation and the introduction of ‘optimal’ or ‘safe’ velocity.

Contrary to this, the original equation of motion of traditional car-following models[8]

$$\ddot{x}_n(t + \tau) = \lambda \{ \dot{x}_{n-1}(t) - \dot{x}_n(t) \} , \quad (4)$$

is essentially a first order differential equation without a delay time  $\tau$  of response. Because traffic flow governed by a first order differential equation is always stable, the delay time  $\tau$  plays a crucial role in describing the behavior of traffic flow. The origin of the delay time  $\tau$  have been thought to be a physiological delay of response. In fact, it is well-known that the motion of a vehicle accompanies some delay time in response to the motion of its preceding vehicle. There seem, however, questions to be discussed: what is the role of the delay time  $\tau$ , or, whether or not the delay time  $\tau$  is equal to the value of observed one.

Here we should remark that it is necessary to distinguish two different types of definition concerning the notion of “delay time”. There is a time lag until the driver begins an action after being conscious of a stimulus. It may also takes a finite time for a vehicle to change its velocity after the operation of driver. We define such physiological and mechanical time lag as “delay time of response”. This can be measured in principle although it may vary depending on the type of the stimulus, individual driver and performance of vehicle. On the other hand, we can define “delay time of vehicle motion” as a period from the time of velocity change of a vehicle to that of following one. The delay time of vehicle motion can also be estimated in observations of real traffic and/or numerical simulations using traffic models. However in general case, it is difficult to find a good definition of the delay time of motion of two successive vehicles. Only in the restricted case in which two vehicles behave quite similarly, for example,

$$v_n(t) \simeq v_{n+1}(t - T), \quad (5)$$

we can define the delay time of motion as  $T$ .

In the car-following models, there are another two types of “delay time” to be distinguished. One is the delay time  $\tau$  explicitly introduced as a parameter in the equation of motion (see Eq.(4)), which we call “explicit delay time” in this paper. This may correspond to the delay time of response. The other is the delay time emerging as a result of the dynamics of traffic flow, which is quite different notion from the explicit delay time. This will correspond to the delay time of vehicle motion stated above. However, this delay time includes not only the contribution from the explicit delay time but also that from purely dynamical origin. We call the latter as “effective delay time”. Obviously, the effective delay time is not zero even if we introduce no explicit delay time.

In this paper, we discuss the delay time of motion in OVM with no explicit delay time. Therefore, because the resulting delay time is equal to the effective delay time, we can investigate the delay time from dynamical origin. This quantity may be compared with observed delay times of vehicle motion in various cases. It will be clear that the effective delay time obtained by our procedure is enough to explain the delay times observed in actual situations of traffic flow.

In section 2, we define the effective delay time of vehicle motion in OVM in terms of the vehicle motions of a leader and its follower and make an analytical study within linear approximation. Then we carry out numerical simulations to obtain the effective delay time in several cases. We show the results in actual traffic situations: the effective delay times under traffic rights (section 3) and those in uniform traffic flow and in congested flow (section 4). Discussions and further prospects are given in section 5.

## 2 Delay Time for Leader and Follower Case

First let us make an definition of effective delay time of vehicle motion. Consider a pair of vehicles, a leader and its follower. This pair of vehicles may be either separate two vehicles or any pair of vehicles picked up from a series of vehicles in highways or from a queue waiting to start under traffic lights.

When a leader moves with its velocity  $v(t)$ , and its follower replicates the motion of the leader with some delay time  $T$  (then the follower's velocity is given by  $v(t - T)$ ), we can define the delay time of vehicle-motion as  $T$ . It must be remarked that we do not define the delay time as the delay of motion of the follower with respect to its position but its velocity replication.

Let the positions of a leader and its follower be  $y(t)$  and  $x(t)$ . In this case Eq.(1) is written as

$$\ddot{x}(t) = a\{V(y(t) - x(t)) - \dot{x}(t)\}. \quad (6)$$

Uniform motion is described by

$$y_0(t) = V(b)t + b, \quad x_0(t) = V(b)t, \quad (7)$$

where  $b$  is headway and  $V(b)$  is a constant velocity. To investigate the response of the follower vehicle to the leader vehicle, we introduce a small perturbation  $\lambda(t)$  and its response  $\xi(t)$ :

$$y(t) = y_0(t) + \lambda(t), \quad x(t) = x_0(t) + \xi(t). \quad (8)$$

Inserting Eq.(8) into Eq.(6) and taking a linear approximation, we get

$$\ddot{\xi}(t) + a\dot{\xi}(t) + af\xi(t) = af\lambda(t), \quad (9)$$

where  $f = V'(b)$ . This is just equivalent to well-known equation of motion for forced oscillation with damping term caused by friction.

In order to find a solution, we first write  $\lambda(t)$  by a Fourier expansion

$$\lambda(t) = \int \tilde{\lambda}(\omega) e^{i\omega t} d\omega. \quad (10)$$

For the Fourier component  $\lambda_0 e^{i\omega t}$ , the solution of Eq.(9) is given by

$$\xi(t) = \frac{\lambda_0}{1 + i\omega/f - \omega^2/af} e^{i\omega t}. \quad (11)$$

This is rewritten as

$$\xi(t) = |\eta| \lambda_0 e^{i\omega(t-T)}, \quad (12)$$

where

$$|\eta|^2 = \frac{|\xi|^2}{\lambda_0^2} = \frac{(af)^2}{(af - \omega^2)^2 + (a\omega)^2}, \quad (13)$$

$$T = \frac{1}{\omega} \tan^{-1} \frac{a\omega}{af - \omega^2}. \quad (14)$$

When  $f < a/2$ , the amplitude  $|\eta|$  is monotonically damping function of  $\omega$ . On the other hand, when  $f > a/2$ ,  $|\eta|$  takes its maximum at  $\omega = \omega_0$ ;

$$\omega_0^2 = a(f - a/2), \quad (15)$$

so we call this  $\omega_0$  as “enhanced mode” (see Fig.2(a)). Note that  $f = a/2$  is the critical point for the instability condition of homogeneous flow as we found in the previous paper [5]. Eq.(15) shows that the enhanced mode  $\omega_0$  ( $\neq 0$ ) exists so far as the instability condition  $f > a/2$  is satisfied.

Let us examine some characteristic cases. For  $f < a/2$  where low frequency modes dominate  $|\omega| \ll a, f$ , we have

$$|\eta| \sim 1, \quad T \sim \frac{1}{f}. \quad (16)$$

In this case the response  $\xi(t)$  to the perturbation  $\lambda(t)$  becomes

$$\xi(t) = \int \tilde{\lambda}(\omega) e^{i\omega(t-T)} d\omega = \lambda(t - T), \quad (17)$$

which leads

$$\dot{x}(t) = V(b) + \dot{\xi}(t) = V(b) + \dot{\lambda}(t - T) = \dot{y}(t - T). \quad (18)$$

Thus for sufficiently slow perturbation, the delay time of vehicle motion becomes  $T$  in Eq.(16), which is approximately the inverse of derivative of the optimal velocity function at corresponding headway.

In the other case  $f > a/2$ , the amplitude  $|\eta|$  takes maximum value at  $\omega = \omega_0$ . Then we have

$$T_{\text{enhanced}} = \frac{1}{\omega_0} \tan^{-1} \frac{2\omega_0}{a}, \quad (19)$$

which indicates that the delay time  $T$  for this enhanced mode depends on the sensitivity  $a$  in contrast to the previous case Eq.(16). One can easily confirm that  $T$  tends to  $1/f$  when  $a$  is close to its critical value  $2f$ . We should remark that an exact replication indicated in Eq.(18) is not realized in this case because  $|\eta|$  is not always equal to 1 in Eq.(12). (See also Figs.1 and the discussions below Eq.(22).)

Now let us see the results of numerical simulations and compare them with those of our analytical consideration. Here and hereafter we use the following form of the optimal velocity function [9],

$$V(\Delta x) = 16.8 [\tanh 0.0860 (\Delta x - 25) + 0.913] \quad (\text{for } \Delta x > 7\text{m}) \quad (20)$$

$$= 0 \quad (\text{for } \Delta x < 7\text{m}) \quad (21)$$

whose parameters are determined from the car-following experiment on Chuo Motorway [10, 11]: the inflection point is  $(\Delta x, \dot{x}) = (25 \text{ m}, 55 \text{ km/h})$ , the maximal velocity is  $V_{\text{max}} = 115 \text{ km/h}$  and the minimal headway is  $\Delta x_{\text{min}} = 7.0 \text{ m}$ , which includes the length of the vehicle (5 m) which was used in the experiment.

In numerical simulations, we prepare a pair of vehicles with their unperturbed motions of Eq.(7) with headway  $b$ . If the leader changes its motion by  $\lambda(t)$  then the function  $\xi(t)$  can be obtained by the numerical simulation from which the delay time can be read off. We choose  $\lambda(t)$  of the leaders motion as

$$y(t) = V(b)t + b - \lambda_0 \cos \omega t, \quad \dot{y}(t) = V(b) + \lambda_0 \omega \sin \omega t \quad \text{for } t \geq 0 \quad (22)$$

with various  $\omega$  setting  $\omega \lambda_0 = 0.1 \text{ m/s}$ , which means that  $|\dot{y}(t) - V(b)| \leq 0.1 \text{ m/s}$ .

As illustrations, we show the behaviors of  $\lambda(t)$  and its response  $\xi(t)$  for  $a = 2.0 \text{ s}^{-1}$ ,  $b = 25 \text{ m}$  (therefore  $\omega_0 = 0.938 \text{ s}^{-1}$ ). Figures 1 are the cases for  $\omega = 0.1, 0.938, 1.5 \text{ s}^{-1}$ . To find the values  $T$ , we first rescale  $\dot{\xi}(t)$  and then translate it so as to coincide the curve  $\dot{\lambda}(t)$  in Figures 1. The value of  $|\eta|$  is this scale factor. The numerical simulations have performed for  $b = 25, 30, 35, 45 \text{ m}$ . In Figures 2, we show the numerical values for  $|\eta|$  and  $T$ . The corresponding analytical results Eqs.(13) and (14) are also shown in these figures. In both figures, the analytical and numerical results agree quite well.

### 3 Delay Time under Traffic Lights

The delay time of vehicle motion is clearly recognized in motions of a series of vehicles under traffic lights. Consider the situation in which every vehicle waits

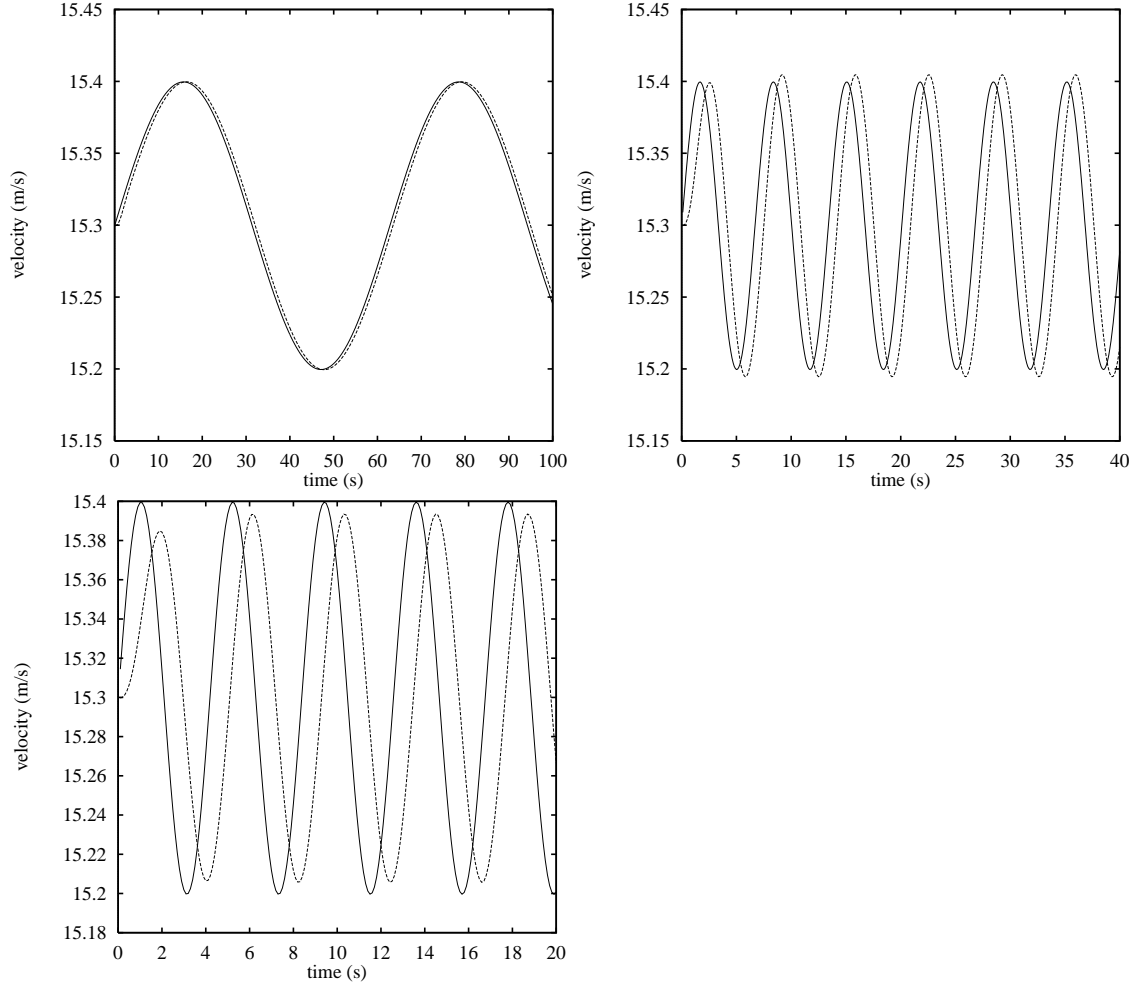


Figure 1: Numerical results for the motion of leader and follower, where  $\Delta x = 25$  m and  $a = 2.0$  s<sup>-1</sup>. Frequencies of leader's motion are (a)  $\omega = 0.1$  s<sup>-1</sup>, (b)  $\omega = \omega_0$  and (c)  $\omega = 1.5$  s<sup>-1</sup>.

until a red light changes to green. The initial conditions are as follows;

$$\begin{aligned} \Delta x_1(0) &= \infty, & \dot{x}_1(0) &= 0 \\ \Delta x_n(0) &= 7(\text{m}), & \dot{x}_n(0) &= 0 \quad (n = 2, 3, \dots) \end{aligned} \quad (23)$$

If the light changes to green, the top vehicle will first accelerates to start, followed by the succeeding vehicles according to the equations of motion.

We perform the numerical simulation for the cases  $a = 2.0, 2.8$  s<sup>-1</sup> and obtain the time dependence of velocity  $\dot{x}_n(t)$  of  $n$ -th vehicles in a queue. Figures 3(a) and 4(a) show the behavior of motion for the first ten vehicles. In the figures, it seems that the behaviors of motion for vehicles on the down stream converge into a common shape. Thus, except first several vehicles, every vehicle in the queue almost replicates the behavior of its preceding one with a certain delay

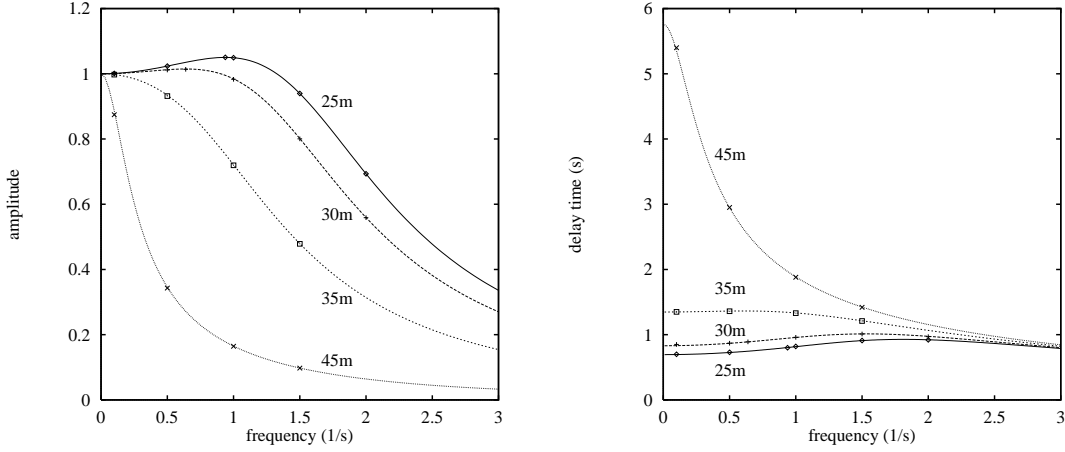


Figure 2: Each curve shows the behavior of (a)  $|\eta|$  (Eq.(13)) and (b)  $T$  (Eq.(14)) for  $b = 25, 30, 35, 45$  m with  $a = 2.0 \text{ s}^{-1}$ . Plotted marks on the curves show numerical results.

time. In this case, the relation (5) can be applied for  $n$ -th vehicle with enough large number  $n$ . In this sense we may say that replicative pattern of vehicle motion is realized “asymptotically” and only in this occasion we can define a delay time of motion  $T$  in just the same way as we defined in section 2. Note that this definition of the delay time slightly differs from a time lag with which the successive vehicles start for example.

From Figures 3(a) and 4(a), we can estimate the delay time  $T = 1.10 \text{ s}$  for  $a = 2.0 \text{ s}^{-1}$  and  $T = 1.03 \text{ s}$  for  $a = 2.8 \text{ s}^{-1}$ . To confirm this similarity of vehicle motion, we also show plots of translated data  $\dot{x}_n(t - (n - 10)T)$  for the 7th, 8th, 9th and 10th vehicles in Figures 3(b) and 4(b).

## 4 Delay Time in Highway Traffic Flow

Next we investigate delay time under a simple situation where  $N$  vehicles move on a single lane circuit with circumference  $L$ . Of course we assume that road conditions are uniform along the circuit and drivers are identical. Numerical calculations are made with the initial condition:  $x_n = 2\delta_{n1} - b(n-1)$  ( $1 \leq n \leq N$ ),  $\dot{x}_n = V(b)$ . The first term of the right hand side of this condition means that a small perturbation is added to the first vehicle  $x_1$ .

In the previous papers [5], we have shown a homogeneous flow changes into congested flow spontaneously if the density of vehicles is greater than the critical value. The results of simulations indicate that after enough time the traffic flow on a circuit creates an alternating pattern of high and low density regions. The motion of vehicles in this flow is visualized by plotting them in the ‘phase space’  $(\Delta x, \dot{x})$ . After the traffic flow becomes stationary, the trajectory of every vehicle



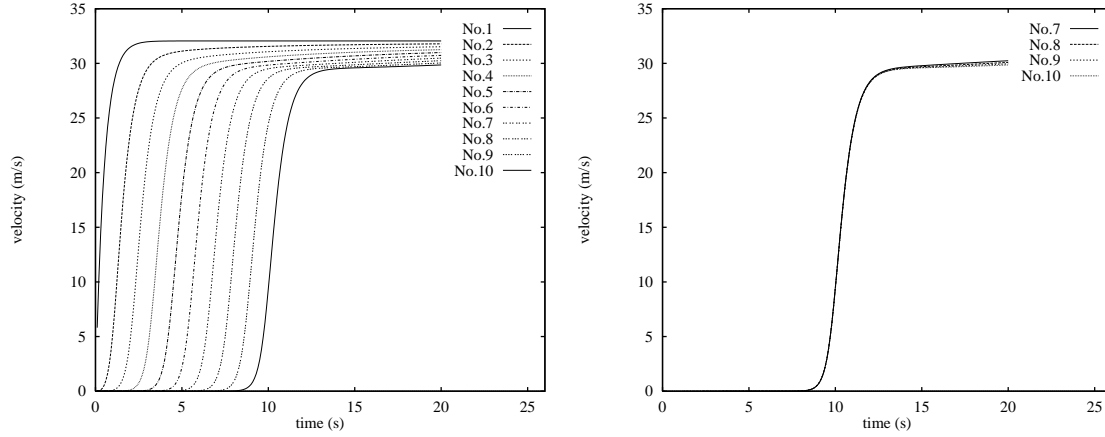


Figure 3: (a) Behavior of velocities of first ten vehicles under traffic lights with  $a = 2.0 \text{ s}^{-1}$ . (b) Figure of shifted curves  $\dot{x}_n(t - (n - 10)T)$  with  $T = 1.10 \text{ s}$ .

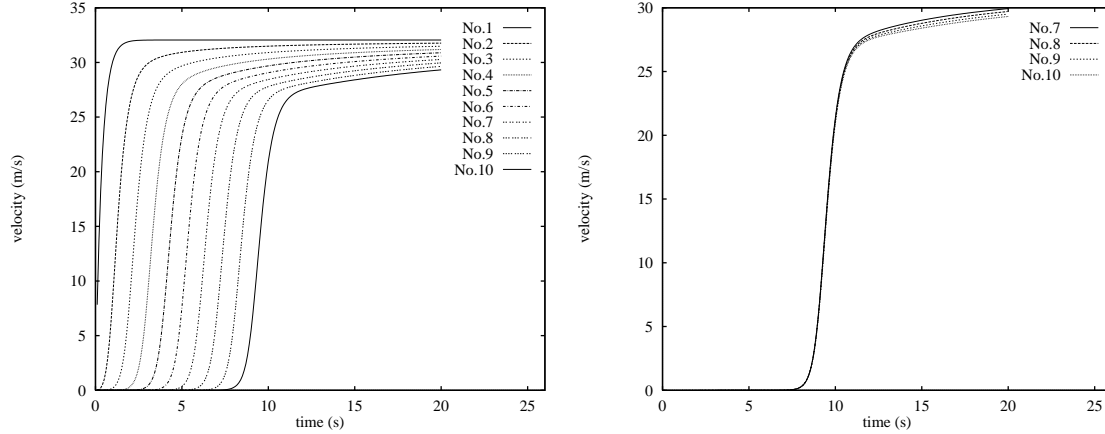


Figure 4: (a) Behavior of velocities of first ten vehicles under traffic lights with  $a = 2.8 \text{ s}^{-1}$ . (b) Figure of shifted curves  $\dot{x}_n(t - (n - 10)T)$  with  $T = 1.03 \text{ s}$ .

in this ‘phase space’ draws a kind of limit cycle which we named ‘hysteresis loop’ in Ref.[5] (see also Fig. 7).

Now let us estimate delay times for two cases under this traffic flow: (A) the first stage and (B) the final stationary-state stage.

• Case A

In this case the traffic flow is almost homogeneous. Let us pick up a pair of vehicles  $n = 10, 11$ . A small perturbation of the first vehicle  $x_1$  propagates backward and after several seconds those pair of vehicles change their velocities. The typical behaviors are demonstrated in Figures 5. In the same way as section 2, the delay time of motion can be estimated from numerical results. The obtained values of delay times are shown in Table 1 for the case of  $a = 2.0 \text{ s}^{-1}$  and Table 2 for the case of  $a = 2.8 \text{ s}^{-1}$ . As references, the delay times for low frequency limit

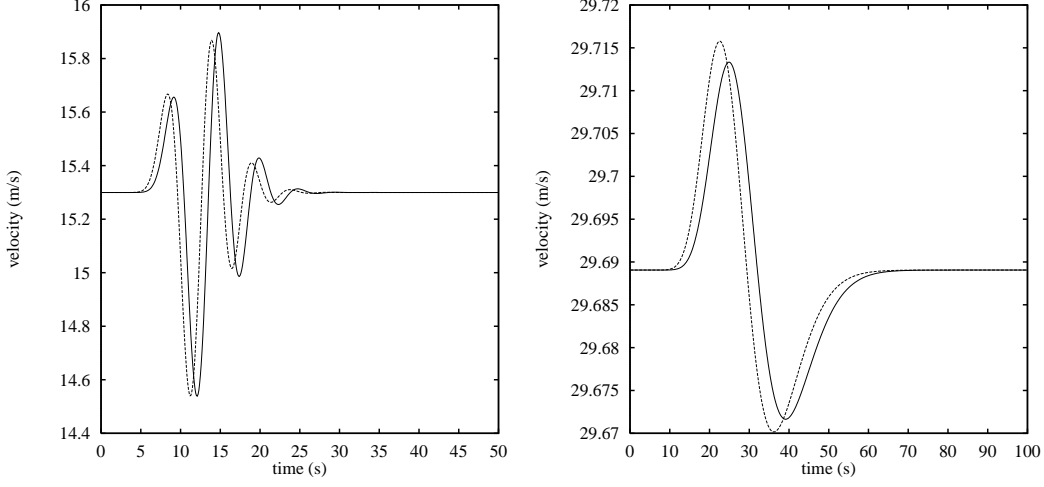


Figure 5: Motions of 10th and 11th vehicles in the first stage: (a)  $\Delta x = 25$  m for 50 seconds and (b)  $\Delta x = 40$  m for 100 seconds with  $a = 2.0 \text{ s}^{-1}$ .

and for the enhanced mode  $\omega_0$  are also shown in Tables 1, 2. From these results, the low frequency limit is a good approximation and the delay time  $T$  is almost independent on sensitivity  $a$  for stable traffic flow. For unstable case, there exist contributions from blow-up modes and they have sensitivity dependence.

• Case B

After sufficiently long time, traffic flow forms stationary patterns of high and low density regions. Under this situation vehicle does not change its velocity unless it encounters a boundary of high and low density regions. Let us observe the motion of a vehicle on the boundary. A vehicle which encounters the boundary changes its velocity. After some delay time, the following vehicle comes to the boundary and changes its velocity in the same way as the previous vehicle. The typical behavior of vehicles is shown in Figure 6.

The delay time of vehicle motion in this case can be derived as follows. Consider two vehicles: one enters into a congested region from a free region and after a certain interval  $T$ , the next one follows. In the ‘phase space’  $(\Delta x, \dot{x})$  (Fig.7), the free region is denoted as a point  $F(\Delta x_F, v_F)$ , that is, vehicles is moving with velocity  $v_F$  and headway  $\Delta x_F$ . The delay time  $T$  is defined as the time which is needed for the next vehicle to reach the boundary and enter into a congested region. Now at the time when the first vehicle reaches the boundary of the free region the distance between the next vehicle and this boundary is of course  $\Delta x_F$ . The next vehicle runs with velocity  $v_F$  and the boundary itself also is moving backward with velocity  $v_B$ , so if the vehicle and the boundary will meet after time interval  $T$ , we can write the following relation

$$v_F T + v_B T = \Delta x_F . \quad (24)$$

Similar relation can be written at a boundary where a vehicle exits from congested

Table 1: Delay times for various headway with  $a = 2.0 \text{ s}^{-1}$ . The second column indicates that traffic flow is stable (-) or unstable (+). The third and fourth columns show analytical results given in section 2.

$\Delta x$ (m)	$f - a/2$	$T_0 = f^{-1}$ (s)	$T_{\text{enhanced}}$ (s)	$T_{\text{simulation}}$ (s)
10	—	2.6427	-	2.6
15	—	1.3434	-	1.35
20	+	0.8282	0.8884	0.95
25	+	0.6921	0.8017	0.85
30	+	0.8282	0.8884	0.95
35	—	1.3434	-	1.35
40	—	2.6427	-	2.6
50	—	13.101	-	13

Table 2: Delay times for various headway with  $a = 2.8 \text{ s}^{-1}$ . The second column indicates that traffic flow is stable (-) or unstable (+). The third and fourth columns show analytical results given in section 2.

$\Delta x$ (m)	$f - a/2$	$T_0 = f^{-1}$ (s)	$T_{\text{enhanced}}$ (s)	$T_{\text{simulation}}$ (s)
10	—	2.6427	-	2.6
15	—	1.3434	-	1.35
20	—	0.8282	-	0.85
25	+	0.6921	0.6996	0.75
30	—	0.8282	-	0.85
35	—	1.3434	-	1.35
40	—	2.6427	-	2.6
50	—	13.101	-	13

region.

$$v_C T + v_B T = \Delta x_C . \quad (25)$$

This can be confirmed if one recalls that the pattern of the flow is already stationary, and the input vehicles of a boundary of congested region must be equal to the output of another boundary. The above two equations, (24) and (25) indicate that the time interval  $T$  and  $v_B$  can be graphically expressed as the slope and the intercept with vertical axis of the line connecting F and C for  $a = 2.0 \text{ s}^{-1}$  and F' and C' for  $a = 2.8 \text{ s}^{-1}$  in Figure 7. If we write the velocity of the first vehicle as  $v(t)$ , then the one of the following vehicle is  $v(t - T)$ , which implies that  $T$  is just the delay time of vehicle-motion defined in the previous section.

It may be convenient to adopt the coordinate moving with the congestion pattern. Let  $x(t)$  and  $X(t)$  be the positions of a vehicle measured in a fixed and

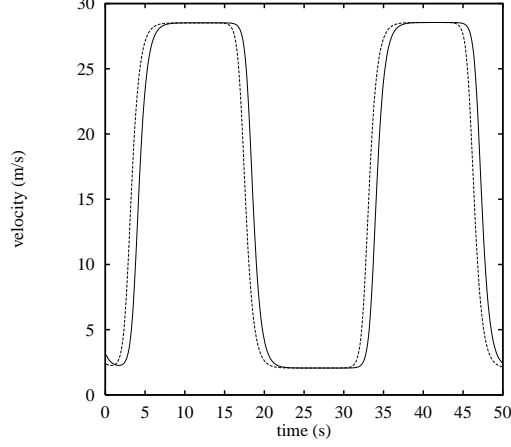


Figure 6: Motion of successive two vehicles in congested flow. Initial condition is  $\Delta x = 25$  m with  $a = 2.0 \text{ s}^{-1}$ .

a moving (with constant velocity  $v_B$ ) coordinates, respectively;

$$x(t) = X(t) + v_B t. \quad (26)$$

Then we get

$$\begin{aligned} \ddot{X}(t) &= a\{V(\Delta X(t)) - (\dot{X}(t) + v_B)\} \\ &= a\{W(\Delta X(t)) - \dot{X}(t)\}, \end{aligned} \quad (27)$$

where  $W(\Delta x)$  is the optimal velocity function in the co-moving frame;

$$W(\Delta x) = V(\Delta x) - v_B. \quad (28)$$

Let us concentrate on the motion of vehicles in this co-moving coordinate. Every vehicle passes a same position with a certain time intervals  $T$ ; in high density region, the time interval is given by

$$T_C = \left( \frac{\Delta X}{\dot{X}} \right)_C, \quad (29)$$

and (similarly) on the point  $F$ , we have,

$$T_F = \left( \frac{\Delta X}{\dot{X}} \right)_F. \quad (30)$$

These two time intervals should be equal since otherwise the congestion pattern moves. We write this time interval by  $T$ ;

$$T = T_C = T_F, \quad (31)$$

which is of course equivalent to Eqs.(24) and (25).

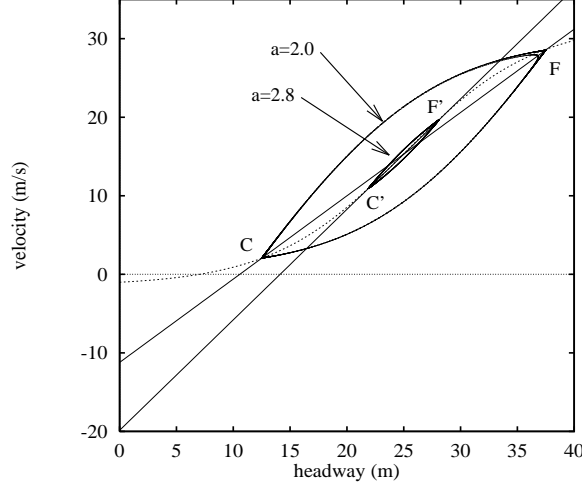


Figure 7: The limit cycles for  $a = 2.0 \text{ s}^{-1}$  and  $a = 2.8 \text{ s}^{-1}$ .

We have carried out numerical simulations. For sensitivity  $a = 2.0 \text{ s}^{-1}$ , we find  $C(12.51, 2.05)$  and  $F(37.50, 28.55)$  yielding the delay of vehicle motion  $T = 0.943 \text{ s}$  and the back velocity of the boundary  $v_B = 11.2 \text{ m/s}$ . As for  $a = 2.8 \text{ s}^{-1}$ ,  $C'(21.89, 10.92)$  and  $F'(28.11, 19.68)$  yielding  $T = 0.711 \text{ s}$  and  $v_B = 19.9 \text{ m/s}$ .

It is interesting to find that the main contribution of the resultant delay of vehicle-motion comes from the structure of the Optimal Velocity Model and not from the explicit delay  $\tau$ .

## 5 Summary and Discussions

The notion of delay time of response  $\tau$  has played a significant role in the history of traffic dynamics. Indeed delays of vehicle motions are observed in many cases, in traffic lights waiting queues or in highway traffic motions and the delay time are usually observed to be of order 1 second. We should take account of the effect of observed delay time and it has long been thought that it must be introduced in the equation of motion as an explicit delay time, most of which is caused by driver's physiological delay time and mechanical delay of response of vehicles. However it is known that the physiological response time is of order 0.1 second, not of order 1 second. We should be careful that the delay time of vehicle motion comes from another origin, that is, from the equation of motion itself which we have here investigated intensively. The results are summarized as follows:

1. The case of a leader vehicle and its follower

As is seen in Figure 2(b), if the headway distance is around 25 m in which drivers are sensitive to the behavior of the motion of the preceding vehicle, we clearly recognize delay time is around 1 second independently of the

frequency of the leader's velocity-change function  $\lambda(t)$ . However if their headway distance is more than 40 m, delay time is estimated to be larger than 1 second, and sometimes we obtain 6 second for the case  $\Delta x = 6$  m for the case of low frequency ( $\omega \sim 0$ ). This is because of the structure of optical velocity function. If the slope of the optimal velocity function is very small, drivers are insensitive to the behavior of the preceding vehicle. This can be easily understood if one considers the extreme case in which the function  $V$  is independent of  $\Delta x$  (and so  $f = 0$ ). In this case, a follower never reacts to its previous vehicle and accordingly its delay time of motion becomes infinity.

2. A queue of vehicles controlled by traffic lights

In this case except the first several vehicles, most of the succeeding vehicles behave almost similarly as seen in Figures 3 and 4. From those figures, delay times are read off;  $T = 1.10$  s for  $a = 2.0$  s<sup>-1</sup> and  $T = 1.03$  s for  $a = 2.0$  s<sup>-1</sup>. Although  $T$  depends on the sensitivity adopted, the results obtained is again of order 1 second for reasonable realistic sensitivity.

3. Many-vehicle case of highway traffic flow

In Figures 5 we show the typical behaviors of a pair of vehicles and in Table 1 and 2 delay times obtained by numerical simulations are summarized with various values of its headway  $\Delta x$ . Again in the case of  $\Delta x = 25$  m, the estimated delay time is found to be of order 1 second, and for the larger  $\Delta x$ , the larger the value of delay time is obtained. In the case where the congested flow becomes stable,  $T = 0.71$  s for  $a = 2.8$  s<sup>-1</sup> and  $T = 0.9471$  s for  $a = 2.0$  s<sup>-1</sup>. Since the end point C (F) on the limit cycle becomes larger (smaller) as  $a$  becomes larger (see for example Fig.7 in ref.[9]), so of course  $T$  becomes smaller for larger  $a$  (high sensitivity).

All our results show that the estimated delay time in our OVM is almost enough to reproduce the order of observed delay time. It now becomes obvious that the delay time of motion arises as an effect of dynamical equation itself without any explicit introduction of  $\tau$ . It is interesting to find the main contribution of the resultant delay of vehicle motion are of same order of magnitudes in any cases. This may come from the structure of optical velocity function itself which we have determined phenomenologically. However we believe that this remarkable fact has its more profound reason, which will be made clearer by further investigation on structure of OVM by performing analytical study.

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